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Anomaly-Free Tensor-Yang-Mills System and Its Dual Formulation

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ABSTRACT

We consider the $(1, 0)$ supersymmetric Yang-Mills multiplet coupled to a self-dual tensor multiplet in six dimensions. It is shown that the counterterm required to cancel the one-loop gauge anomaly modifies the classical equations of motion previously obtained by Bergshoeff, Sezgin and Sokatchev (BSS). We discuss the supermultiplet structure of the anomalies exhibited in the resulting equations of motion. The anomaly corrected field equations agree with the global limit, recently obtained by Duff, Liu, Lu and Pope, of a matter coupled supergravity theory in six dimensions. We also obtain the dual formulation of the BSS model in which the tensor multiplet is free while the field equations of the Yang-Mills multiplet contain the fields of the tensor multiplet at the classical level.

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1 Introduction

Globally supersymmetric field theories in various dimensions have recently been playing new and interesting rôles in the context of the physics of branes. There is now a large amount of literature devoted to this subject which we shall not attempt to review here, even briefly. It suffices to mention that such theories, or suitably nonlinearised variations of them, are candidate worldvolume theories, at least in principle, for superbranes. This consideration motivated Bergshoeff, Sokatchev and one of the authors [1] to construct the coupling of an $N = (1, 0)$ tensor multiplet to Yang-Mills in six dimensions (see [1] for a more detailed discussion of other motivations). The construction was a direct one not employing global limits of matter coupled $D = 6$ supergravity theories known at the time [2, 3]. In fact, it was not clear in [1] how to take such limits. A peculiar aspect of the BSS model is that while the field equations of the tensor multiplet contain the Yang-Mills field, the reverse is not the case. Nevertheless, the model is perfectly gauge invariant and supersymmetric at the classical level.

In a recent paper, Duff, Liu, Lu and Pope (DLLP) [4] found a way to take the global limit of $D = 6$ supergravity coupled to Yang-Mills and multi-tensor multiplets [5, 6, 7, 8, 9, 10] and indeed recovered the BSS model by this means. However, DLLP also considered a limit in which the Yang-Mills field equations do contain the tensor multiplet fields. (See also [10] for a discussion of superspace constraints of supergravity plus Yang-Mills system giving rise to the BSS model in a particular limit).

The main purpose of this paper is to show that the DLLP global limit [4] corresponds to the BSS model augmented with terms that originate from the Green-Schwarz style anomaly-cancelling local counterterms, and to exhibit the structure of what is known as the anomaly supermultiplet involved in this case [11, 12]. Analogous considerations have arisen in the case of supergravity plus matter systems in [7] and in more detail in [8].

We mentioned above that a peculiar aspect of the BSS model is that while the field equations of the tensor multiplet contain the Yang-Mills field, the reverse is not the case. In this paper, we will show that the BSS model has a dual formulation in which the opposite situation arises, namely while the tensor multiplet is free, the field equations for the Yang-Mills multiplet contain the tensor multiplet fields! This is achieved by reversing the role of the dimensionful parameter α' that arises in the BSS model and the Planck constant \hbar that arises when one adds the anomaly cancelling terms. The classical dual model is perfectly gauge invariant, and the \hbar dependent corrections to the equations which make the tensor multiplet interacting arise at the quantum level as a result of the anomaly cancellation mechanism.

After recalling the BSS model in the next section we determine the anomaly-corrected field equations via the anomaly supermultiplet in the subsequent sections, both in superspace and in the component formalism. The dual formulation of the BSS model is described in the penultimate section. Further comments on the anomaly-corrected version of the BSS model are given in the conclusions.

2 The Classical Tensor-Yang-Mills System

Consider $(1, 0)$ superspace in $D = 6$ with coordinates $z^M = (x^\mu, \theta^{\alpha i})$ where $\theta^{\alpha i}$ are symplectic Majorana-Weyl spinors carrying the $Sp(1)$ doublet index $i = 1, 2$. The basic superfields we shall consider are the supervielbein E_M^A , the super two-form $B = \frac{1}{2!} dz^M \wedge dz^N B_{NM}$ and the Lie algebra valued Yang-Mills super one-form $A = dz^M A_M$. (Our conventions for super p -forms are as in [15]). The torsion super two-form T^A , the Yang-Mills curvature two-form F and the Chern-Simons modified super three-form \mathcal{H} are defined as follows

$$\begin{aligned} T^A &= dE^A, \quad F = dA + A \wedge A, \\ \mathcal{H} &= \frac{1}{2} dz^M dz^N dz^P \left(\partial_P B_{NM} - \frac{\alpha'}{2} \text{tr}(A_P F_{NM} - \frac{2}{3} A_P A_N A_M) \right). \end{aligned} \quad (1)$$

They satisfy the (flat) superspace Bianchi identities $dT^A = 0$, $DF = 0$ and

$$d\mathcal{H} = \frac{\alpha'}{8} \text{tr} F \wedge F, \quad (2)$$

where α' is an arbitrary dimensionful constant and $D = d + A$. In flat $D = 6$ superspace the only nonvanishing torsion component is

$$T_{\alpha i, \beta j}{}^a = 2\varepsilon_{ij}(\gamma^a)_{\alpha\beta}. \quad (3)$$

Next, we introduce the Lie algebra-valued spinor superfield $\lambda_{\alpha i}$ and a scalar superfield ϕ . The BSS model is then characterised by the constraints [1]

$$\begin{aligned} F_{\alpha i, \beta j} &= 0, \quad F_{a\alpha i} = -(\gamma_a)_{\alpha\beta} \lambda_i^\beta, \\ \mathcal{H}_{ab\alpha i} &= 0, \quad \mathcal{H}_{\alpha i, \beta j, \gamma k} = 0, \\ \mathcal{H}_{a\alpha i, \beta j} &= -2\phi(\gamma_a)_{\alpha\beta} \varepsilon_{ij}, \\ \mathcal{H}_{ab\alpha i} &= -(\gamma_{ab})_\alpha{}^\beta D_{\beta i} \phi, \\ \mathcal{H}_{abc}^+ &= (\gamma_{abc})^{\alpha\beta} D_\alpha^i D_{\beta i} \phi, \\ \mathcal{H}_{abc}^- &= \alpha'(\gamma_{abc})_{\alpha\beta} \text{tr} \lambda^{\alpha i} \lambda_i^\beta, \\ D_\alpha^{(i} D_{\beta}^{j)} \phi &= \alpha' \epsilon_{\alpha\beta\gamma\delta} \text{tr} \lambda^{\gamma(i} \lambda^{\delta j)}. \end{aligned} \quad (4)$$

where \mathcal{H}_{abc}^- is anti-self-dual projected and \mathcal{H}_{abc}^+ is self-dual projected, i.e. $\mathcal{H}_{abc}^\pm = 1/2 (\mathcal{H}_{abc} \pm \tilde{\mathcal{H}}_{abc})$. The components of the gauge spinor superfield $\lambda_{\alpha i}$ can be defined as

$$\lambda^{\alpha i} = \lambda^{\alpha i}|, \quad F^{ab} = (\gamma^{ab})^\alpha_\beta D_{\alpha i} \lambda^{\beta i}|, \quad Y^{ij} = D_\alpha^{(i} \lambda^{\alpha j)}|, \quad (5)$$

and the components of the tensor multiplet dilaton superfield ϕ as

$$\sigma = \phi|, \quad \chi_{\alpha i} = D_{\alpha i} \phi|, \quad H_{abc}^+ = \gamma_{abc}^{\alpha\beta} D_\alpha^i D_{\beta i} \phi|, \quad (6)$$

where $H = dB$ and the vertical bar indicates evaluation of a superfield at $\theta = 0$. Using the information provided above, one then finds the component form of the field equations [1]

$$\begin{aligned} \mathcal{H}_{\mu\nu\rho}^- &= -\frac{\alpha'}{2} \text{tr} (\bar{\lambda} \gamma_{\mu\nu\rho} \lambda), \\ \gamma^\mu \partial_\mu \chi^i &= \alpha' \text{tr} \left(\frac{1}{4} \gamma^{\mu\nu} F_{\mu\nu} \lambda^i + Y^{ij} \lambda_j \right), \\ \partial_\mu \partial^\mu \sigma &= \alpha' \text{tr} \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - 2 \bar{\lambda} \gamma^\mu D_\mu \lambda + Y^{ij} Y_{ij} \right), \end{aligned} \quad (7)$$

where $D\lambda = d\lambda + [A, \lambda]$. The first equation can be rewritten as

$$\partial_\eta \mathcal{H}^{+\eta\lambda\tau} = \frac{\alpha'}{48} \varepsilon^{\lambda\tau\mu\nu\rho\sigma} \text{tr} \left(F_{\mu\nu} F_{\rho\sigma} - \frac{4}{3} \bar{\lambda} \gamma_{\mu\nu\rho} D_\sigma \lambda \right). \quad (8)$$

The supersymmetry transformations of the system are ²

$$\begin{aligned} \delta A_\mu &= -\bar{\epsilon} \gamma_\mu \lambda, \\ \delta \lambda^i &= \frac{1}{8} \gamma^{\mu\nu} F_{\mu\nu} \epsilon^i - \frac{1}{2} Y^{ij} \epsilon_j, \\ \delta Y^{ij} &= -\bar{\epsilon}^{(i} \gamma^\mu D_\mu \lambda^{j)}. \end{aligned} \quad (9)$$

The corresponding rules for the on-shell self-dual tensor multiplet coupled to Yang–Mills are

$$\begin{aligned} \delta \sigma &= \bar{\epsilon} \chi, \\ \delta \chi^i &= \frac{1}{48} \mathcal{H}_{\mu\nu\rho}^+ \gamma^{\mu\nu\rho} \epsilon^i + \frac{1}{4} (\partial_\mu \sigma) \gamma^\mu \epsilon^i - \frac{\alpha'}{4} \text{tr} (\gamma^\mu \lambda^i \bar{\epsilon} \gamma_\mu \lambda), \\ \delta B_{\mu\nu} &= \alpha' \text{tr} A_{[\mu} \delta A_{\nu]} - \bar{\epsilon} \gamma_{\mu\nu} \chi, \end{aligned} \quad (10)$$

The commutator of two supersymmetry transformations (10) closes on all components of the tensor multiplet modulo the field equations (7), and the supersymmetry algebra is [1]

²We use the notation and conventions of [16]. In particular, (A, λ, Y_{ij}) take values in the Lie algebra of the corresponding gauge group and the contraction of $Sp(1)$ indices in fermionic bilinears is suppressed.

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta(\xi^a) + \delta(\Lambda) + \delta(\Lambda_a) , \quad (11)$$

where the translation parameter ξ^a , the tensor gauge transformation parameter Λ_a and the gauge parameter Λ are given by

$$\xi^a = \frac{1}{2}\bar{\epsilon}_2\gamma^a\epsilon_1 , \quad \Lambda_a = \xi^b B_{ba} + \sigma\xi_a , \quad \Lambda = -\xi^a\Lambda_a , \quad (12)$$

and the tensor gauge transformation takes the form

$$\delta_\Lambda B_{\mu\nu} = -\frac{\alpha'}{2}\text{tr } \Lambda(\partial_\mu A_\nu - \partial_\nu A_\mu) . \quad (13)$$

The tensor multiplet is always on-shell (in fact, its off-shell version is not known), but the Yang-Mills multiplet is still off-shell. It is put on-shell by setting the triplet of auxiliary fields to zero,

$$Y_{ij} = 0 , \quad (14)$$

which then implies the Yang-Mills equations of motion

$$\begin{aligned} \gamma^\mu D_\mu \lambda &= 0 , \\ D_\mu F^{\mu\nu} + 2[\bar{\lambda}, \gamma^\nu \lambda] &= 0 . \end{aligned} \quad (15)$$

Note that these equations do not contain the fields of the tensor multiplet. Despite this odd feature, the system of equations of motion (7) and (15) are perfectly consistent in that they do possess classical gauge symmetry and supersymmetry. At the quantum level, the system develops gauge anomalies due to the chiral gauge fermions circulating in loops. For certain groups these anomalies can be cancelled by the Green-Schwarz mechanism, and in such cases one can derive the effect of the anomaly-cancelling counterterm as we shall see in the next section.

3 The Inclusion of Anomalies

The only possible local anomaly in the model described above is the gauge anomaly due to the minimal coupling of the Yang-Mills field with the chiral gauge fermions. The anomaly polynomial is thus proportional to $(\dim G)\text{tr} F^4$. The associated gauge anomaly can be cancelled by the Green-Schwarz mechanism provided that the anomaly polynomial factorizes as $(\text{tr} F^2)^2$. This factorization is possible only for the gauge groups $E_8, E_7, E_6, F_4, G_2, SU(3), SU(2), U(1)$, or any of their products with each other [17]. Assuming that an anomaly free group is chosen, then the gauge transformation (13) of the antisymmetric tensor field can be utilized to cancel the anomaly as usual. To this end, one would normally introduce a counterterm proportional to

$$\hbar B_2 \wedge \text{tr } F \wedge F , \quad (16)$$

together with its supersymmetry partners in the one-loop effective Lagrangian. According to [19], one of these partner terms is

$$\hbar \, \sigma \text{tr} F^2 . \quad (17)$$

This means that the Yang-Mills gauge coupling is quantum mechanically modified to

$$\frac{1}{g^2(\sigma)} = \frac{1}{g^2} + \hbar \, \sigma . \quad (18)$$

This fact will be used later. We should note, however, that, strictly speaking, one does not have a manifestly Lorentz invariant action (due to the presence of the antisymmetric tensor field B) unless one introduces new fields. As we do not wish to introduce new fields here, we have to proceed in a different manner which nonetheless captures the corrections to the equations of motion at the one-loop level. We shall do this by considering the structure of the supercurrent multiplet for a gauge theory in six dimensions, as this multiplet has the same structure as the Yang-Mills equation of motion multiplet.

On quite general grounds, it has been known for sometime [11] that the supercurrent for a six-dimensional Abelian gauge theory is a linear superfield J_{ij} defined by

$$D_\alpha^{(i} J^{jk)} = 0 , \quad J^{ij} = J^{ji} , \quad (19)$$

Its components are

$$J^{ij} = \{j^{ij}, \eta_\alpha^i, V_\mu\} , \quad (20)$$

where

$$\partial^\mu V_\mu = 0 . \quad (21)$$

Such a current supermultiplet couples to the prepotential for $D = 6$ super-Maxwell theory, $V_{ij} = V_{ji}$. The coupling is

$$\int d^6x d^8\theta J^{ij} V_{ij} . \quad (22)$$

Due to the constraint on the current multiplet, this interaction is invariant under the gauge transformations

$$\delta V_{ij} = D_\alpha^k \Lambda_{ijk}{}^\alpha , \quad (23)$$

where the gauge parameter superfield is totally symmetric in its ijk indices.

If there is an anomaly, the constraint on the current multiplet will be modified to

$$D_\alpha^{(i} J^{jk)} = A_\alpha^{ijk} . \quad (24)$$

The nonvanishing of A_α^{ijk} means that the component current V_μ is no longer conserved. The anomaly superfield A_α^{ijk} defines a possibly reducible supermultiplet which can be called the anomaly supermultiplet [11, 12]. Its precise form depends on the details of the model.

The equation of motion multiplet for super-Maxwell theory has exactly the same structure as the current multiplet as the variation of the action will result in a coupling of the form (22), with V_{ij} replaced by its variation. In the non-Abelian case, the solution of constraints on the superspace field strength in terms of the prepotential V_{ij} (now Lie algebra-valued) is rather complicated [13, 14], but, as far as the equations of motion are concerned, the variation of Yang-Mills action will give rise to an integral of the form

$$\int d^6x d^8\theta \text{Tr}(\delta V_{ij} J^{ij}) . \quad (25)$$

The equation of motion of multiplet, which is itself Lie algebra-valued, now satisfies the non-Abelian conservation constraint

$$\nabla_{\alpha(i} J_{jk)} = 0 \quad (26)$$

In fact, in the classical theory, the equation of motion multiplet is simply the superfield whose leading component is the auxiliary field Y_{ij} . Note that all the components of the equation of motion multiplet are now Lie algebra-valued, in particular, the component V_μ will be covariantly conserved.

To accommodate the anomaly counterterm (16) in this framework we therefore need to amend the equation of motion multiplet by terms proportional to \hbar , so that the covariant conservation condition will be modified by an anomaly term which will now be Lie algebra-valued as well. Thus, in the BSS model, we set

$$J_{ij} = (1 + \hbar \sigma) Y_{ij} - 2\hbar \bar{\lambda}_{(i} \chi_{j)} . \quad (27)$$

This form of the \hbar corrections is based on dimensional considerations, as well as the chirality and $Sp(1)$ representation content of the available spinor fields in the theory. The $\hbar\sigma Y$ term is motivated by the formula (18) (we have set $g = 1$ for simplicity). To see this, one should observe that the second supervariation of (27) will yield an expression which contains in particular the term $D^\mu((1 + \hbar \sigma) F^{\mu\nu})$ which has the required form to be consistent with (18). We shall also see that this result is perfectly in agreement with the global limit of anomaly-free supergravity theory in six dimensions as well [4].

Next we compute the anomaly superfield A_α^{ijk} from (24). Taking the spinorial derivative of J_{ij} defined in (27) amounts to its supervariation. With the aid of the supervariations provided in Sec. 2, one finds that only the \hbar dependent part of (27) gives a nonvanishing contribution (upon symmetrization in ijk) and the result is simply

$$A_{\alpha ijk} = \hbar \alpha' \varepsilon_{\alpha\beta\gamma\delta} \text{tr} \lambda_{(i}^\beta \lambda_j^\gamma \lambda_{k)}^{\delta'} , \quad (28)$$

where $\varepsilon_{\alpha\beta\gamma\delta}$ is the constant totally antisymmetric tensor, and where we have put a prime on one of the gauge fermions to indicate that it is not involved in the trace. We have also used the Fierz identity

$$(\gamma^a)_{\alpha\beta} (\gamma_a)_{\gamma\delta} = 2\varepsilon_{\alpha\beta\gamma\delta}. \quad (29)$$

The λ^3 term is, therefore, the lowest dimensional component of the anomaly supermultiplet. By acting repeatedly with the supercovariant derivative on it, one can obtain all of its components. In particular, one finds the covariant anomaly $F \wedge \text{tr } F \wedge F$ among these components.

The anomaly corrections to the field equations can now be obtained as follows. To begin with, we set

$$J_{ij} = 0, \quad (30)$$

as a consequence of which the Yang-Mills auxiliary field is no longer vanishing but picks up \hbar dependent corrections

$$Y_{ij} = \frac{2\hbar}{1 + \hbar\sigma} \bar{\lambda}_{(i} \chi_{j)} . \quad (31)$$

Next, we observe that the supersymmetric variation of J_{ij} defined in (27) takes the form

$$\delta J_{ij} = \epsilon^{\alpha k} \left(\varepsilon_{k(i} \Lambda_{j)\alpha} + A_{\alpha ijk} \right), \quad (32)$$

where $A_{\alpha ijk}$ is the anomaly as given in (28) and where, suppressing the indices of the spinors,

$$\begin{aligned} \Lambda_i = & - (1 + \hbar\sigma) \gamma^\mu D_\mu \lambda_i + \frac{1}{24} \hbar \mathcal{H}_{\mu\nu\rho}^+ \gamma^{\mu\nu\rho} \lambda_i - \frac{1}{2} \hbar (\partial_\mu \sigma) \gamma^\mu \lambda_i + \frac{1}{4} \hbar F_{\mu\nu} \gamma^{\mu\nu} \chi_i \\ & - \frac{2\hbar^2}{1 + \hbar\sigma} \bar{\lambda}_{(i} \chi_{j)} \chi^j - \frac{1}{4} \alpha' \hbar \text{tr} (\gamma^\mu \lambda_i \bar{\lambda} \gamma_\mu \lambda') . \end{aligned} \quad (33)$$

The prime on a field means that it is not involved in a trace. The on-shell equation (27) has been used in obtaining this result. Upon total symmetrization in ijk of the terms in the parenthesis in (32), the first term drops out and the second term yields the anomaly (28). Thus, $J_{ij} = 0$ is a superfield equation modulo anomalies. Accordingly, isolating this anomaly term in the variation of J_{ij} , we can interpret

$$\Lambda_{i\alpha} = 0 \quad (34)$$

as the gauge fermion field equation. From (33), we thus obtain the anomaly-corrected gauge fermion field equation:

$$\begin{aligned} (1 + \hbar\sigma) \gamma^\mu D_\mu \lambda_i = & \frac{1}{24} \hbar \mathcal{H}_{\mu\nu\rho}^+ \gamma^{\mu\nu\rho} \lambda_i - \frac{1}{2} \hbar (\partial_\mu \sigma) \gamma^\mu \lambda_i + \frac{1}{4} \hbar F_{\mu\nu} \gamma^{\mu\nu} \chi_i \\ & - \frac{2\hbar^2}{1 + \hbar\sigma} \bar{\lambda}_{(i} \chi_{j)} \chi^j - \frac{1}{4} \alpha' \hbar \text{tr} (\gamma^\mu \lambda_i \bar{\lambda} \gamma_\mu \lambda') . \end{aligned} \quad (35)$$

It is gratifying to observe that the field equation (35) agrees with the global limit of [4] ³. Thus, there really are not two different global limits of the anomaly-free 6D supergravity plus Yang-Mills system, but only one such limit, with or without anomaly corrections proportional to \hbar .

There is an arbitrariness in the coefficient of the last term in (35) [8] which is related to the anomalies in the system. To see this, observe that the transformation rule for the Yang-Mills auxiliary field Y_{ij} as calculated from (31) differs from the result obtained from (9) when one uses the anomaly modified gauge fermion field equation (35) by $\alpha'\hbar\lambda^3$ terms. This means that the closure of the supersymmetry algebra is spoiled by anomalies, which is hardly surprising.

The transformation rule for Y_{ij} can be modified so that it agrees with the transformation rule obtained from (31), provided that the $\alpha'\hbar$ dependent last term in the gauge fermion equation (35) is removed. The required modification is

$$\delta'Y_{ij} = -\frac{\alpha'\hbar}{2(1+\hbar\sigma)} \bar{e}\gamma_\mu\lambda \bar{\lambda}'_{(i}\gamma^\mu\lambda_{j)} . \quad (36)$$

As a consequence of this modification, the anomaly term in (32) disappears as well but the anomaly continues to manifest itself in the nonclosure of the supersymmetry algebra.

Next, we calculate the Yang-Mills equation of motion from the supersymmetry variation of (35). Again, isolating the terms that cannot be absorbed into the Yang-Mills equation, and therefore belong to the anomaly supermultiplet, we can determine all the \hbar corrections to the Yang-Mills equation. The variation of the gauge fermion equation of motion (35) takes the form

$$\delta\Lambda_i = -\frac{1}{4}\gamma^\mu\epsilon_i V_\mu + \text{anomalous terms} \quad (37)$$

where the anomalous terms are those which can not be absorbed into V_μ and

$$\begin{aligned} V^\nu &= D_\mu [(1+\hbar\sigma)F^{\mu\nu}] + \frac{1}{2}\hbar \mathcal{H}_{\nu\rho\sigma} F^{\rho\sigma} + 2(1+\hbar\sigma)[\bar{\lambda}, \gamma^\nu\lambda] \\ &\quad + 2\hbar D_\mu (\bar{\chi}\gamma^{\mu\nu}\lambda) - \frac{\alpha'\hbar}{2} \text{tr} \left(\bar{\lambda}\gamma^{\nu\rho\sigma}\lambda F'_{\rho\sigma} \right) \end{aligned} \quad (38)$$

up to ambiguous $\alpha'\hbar\lambda^2 F$ and $\alpha'\hbar^2\lambda^3\chi$ terms (more on this below). Thus, setting

$$V^\mu = 0 , \quad (39)$$

gives the anomaly-corrected Yang-Mills field equation

$$\begin{aligned} D_\mu [(1+\hbar\sigma)F^{\mu\nu}] &= -\frac{1}{2}\hbar \mathcal{H}_{\nu\rho\sigma} F^{\rho\sigma} - 2(1+\hbar\sigma)[\bar{\lambda}, \gamma^\nu\lambda] \\ &\quad - 2\hbar D_\mu (\bar{\chi}\gamma^{\mu\nu}\lambda) + \frac{\alpha'\hbar}{2} \text{tr} \left(\bar{\lambda}\gamma^{\nu\rho\sigma}\lambda F'_{\rho\sigma} \right) . \end{aligned} \quad (40)$$

³In comparing the cubic fermion terms, it is useful to note the Fierz identities: $\bar{\lambda}^{(i}\chi^{j)}\lambda_j = \frac{3}{4}\bar{\chi}\lambda\lambda^i - \frac{1}{8}\bar{\chi}\gamma_{\mu\nu}\lambda\gamma^{\mu\nu}\lambda^i$ and $\bar{\lambda}^{(i}\chi^{j)}\chi_j = -\frac{3}{4}\bar{\lambda}\chi\chi^i + \frac{1}{8}\bar{\lambda}\gamma_{\mu\nu}\chi\gamma^{\mu\nu}\chi^i$.

The purely bosonic part of this result agrees with the global limit of [4] where the bosonic part of the Yang-Mills equation is considered. The global limit of the full supergravity coupled to tensor plus Yang-Mills system is expected to give the fermionic terms in (40). The tensor multiplet equations are given in (7), with Y_{ij} replaced by the fermionic bilinear form given in (27). The result agrees precisely with the global limit of [4].

The α'/\hbar dependent terms in (40) are ambiguous due to the possible shifting of anomalies to the supersymmetry transformation rules. The ones exhibited in (40) are such that the divergence of the Yang-Mills current is purely the covariant anomaly, namely

$$D_\mu V^\mu = -\frac{1}{16}\alpha'\hbar \epsilon^{\mu\nu\rho\sigma\lambda\tau} \text{tr} (F_{\mu\nu}F_{\rho\sigma}F'_{\lambda\tau}) . \quad (41)$$

We conclude this section by commenting on the supermultiplet structure of the anomalies. The equations of motion form an $8 + 8$ component linear multiplet in the absence of anomalies. To include the effect the anomalies, one has to enlarge the multiplet to accommodate the field $A_{\alpha ijk}$ defined in (28). At first sight, a candidate such multiplet is the $32 + 32$ component “relaxed hypermultiplet” constructed sometime ago in [21, 20] (this is somewhat of a misnomer; it would be more appropriate to call it the “relaxed linear multiplet”). This multiplet contains the $8 + 8$ equations of motion components $(J_{ij}, \Lambda_{\alpha i}, V_\mu)$ and the $24 + 24$ anomaly components $(L_{ijkl}, A_{\alpha ijk}, V_{\mu ij}, \xi_{\alpha i}, C)$, where the $Sp(1)$ indices are symmetrized. The supersymmetry transformation rules for all these fields can be found in [20]. In particular, the variation of J_{ij} is seen to agree with (32). Comparing the transformation (37) with those of [20], one finds that the V_μ term is in agreement and that the anomalous terms in (37) are candidate for the field $V_{\mu ij}$. However, the supersymmetric variation of $A_{\alpha ijk}$ must yield the field L_{ijkl} in addition to $V_{\mu ij}$ according to the relaxed hypermultiplet transformation rules given in [20]. In the model considered here, the supersymmetry variation of the anomaly $A_{\alpha ijk}$ can yield the $V_{\mu ij}$, but it is not clear if it can yield the field L_{ijkl} . Instead a field of the form $W_{\mu i j k l}$ arises and, as far as we can see, it is not curl free. This suggests that, in order to capture the full supermultiplet structure of the equations of motion plus anomalies, one should relax the “relaxed hypermultiplet” even further, for example by relaxing the (covariant) curl-free condition on $W_{\mu i j k l}$. This point requires further investigation.

4 The Dual Model

We know from old results [22, 23] that the anomaly cancellation in a dual formalism works in such a way that the anomaly polynomial is still the same, but the rôle of the classical counterterm and \hbar dependent gauge transformation get interchanged. Namely, in the dual formulation the one loop level counterterm (16) (proportional to \hbar) becomes a classical term, while the gauge variation (13) (which is classical in the BSS model) arises at the quantum level (proportional to \hbar).

This suggests that one makes the interchange

$$\alpha' \leftrightarrow \hbar \quad (42)$$

in the formulae discussed in this paper as a consequence of which the tensor field equations

now become free, while the Yang-Mills equations have interaction terms proportional to the tensor multiplet. In this case the Chern-Simons modification to \mathcal{H} is proportional to \hbar and consequently, the Yang-Mills equation is anomaly free at the classical level.

The dualization procedure just described yield the following system of classical equations of motion. Firstly, the tensor multiplet equations of motion become

$$\begin{aligned} H_{\mu\nu\rho}^- &= 0 \\ \gamma^\mu \partial_\mu \chi^i &= 0 , \\ \partial_\mu \partial^\mu \sigma &= 0 , \end{aligned} \tag{43}$$

where H no longer contains the Chern-Simons form and is given by $H = dB$. While the tensor multiplet has become free, the equations of motion for the Yang-Mills multiplet now contain the tensor multiplet as follows

$$\begin{aligned} (1 + \alpha' \sigma) \gamma^\mu D_\mu \lambda_i &= \frac{1}{24} \alpha' H_{\mu\nu\rho}^+ \gamma^{\mu\nu\rho} \lambda_i - \frac{1}{2} \alpha' (\partial_\mu \sigma) \gamma^\mu \lambda_i + \frac{1}{4} \alpha' F_{\mu\nu} \gamma^{\mu\nu} \chi_i \\ &\quad - \frac{2\alpha'^2}{1 + \alpha' \sigma} \bar{\lambda}_{(i} \chi_{j)} \chi^j , \end{aligned} \tag{44}$$

$$D_\mu [(1 + \alpha' \sigma) F^{\mu\nu}] = -\frac{1}{2} \alpha' H_{\nu\rho\sigma}^+ F^{\rho\sigma} - 2\alpha' D_\mu (\bar{\chi} \gamma^{\mu\nu} \lambda) - 2(1 + \alpha' \sigma) [\bar{\lambda}, \gamma^\nu \lambda] . \tag{45}$$

The system of equations (44) and 2 are invariant under the free tensor multiplet supersymmetry transformations

$$\begin{aligned} \delta \sigma &= \bar{\epsilon} \chi , \\ \delta \chi^i &= \frac{1}{48} H_{\mu\nu\rho}^+ \gamma^{\mu\nu\rho} \epsilon^i + \frac{1}{4} (\partial_\mu \sigma) \gamma^\mu \epsilon^i , \\ \delta B_{\mu\nu} &= -\bar{\epsilon} \gamma_{\mu\nu} \chi , \end{aligned} \tag{46}$$

and the interacting Yang-Mills multiplet supersymmetry transformations

$$\begin{aligned} \delta A_\mu &= -\bar{\epsilon} \gamma_\mu \lambda , \\ \delta \lambda^i &= \frac{1}{8} \gamma^{\mu\nu} F_{\mu\nu} \epsilon^i - \frac{\alpha'}{1 + \alpha' \sigma} \bar{\lambda}^{(i} \chi^{j)} \epsilon_j . \end{aligned} \tag{47}$$

Note that the terms previously proportional to α' have been dropped since they now correspond to quantum corrections proportional to \hbar . The system of equations given above represent the dual formulation of BSS model. Like the BSS model, it is perfectly consistent at the classical level. One can check, for example, that the Yang-Mills equation (45) is consistent. To show that the divergence of the Yang-Mills equation does not give rise to an anomaly, one uses $dH = 0$ and the equations of motion for the fermions.

5 Comments

The corrections to the equations of motion determined here by anomaly considerations are only those which originate from the Green-Schwarz anomaly cancelling *local* counterterms. To obtain the fully consistent equations of motion at the quantum level one must also take into account the non-local corrections to the one loop effective action. This raises the question of which equations of motion are to be solved in search of special solutions of the theory. It is clearly necessary to check that the anomalous terms in the equations of motion, e.g. the anomalous divergence of the Yang-Mills equation, vanish for a given solution. However, this may not be sufficient. One should in principle check that the remaining part of the one-loop effective action does not spoil the solution.

In $D = 10$ where one also has a supergravity plus Yang-Mills system with Green-Schwarz anomaly cancellation mechanism, one usually appeals to an underlying and exactly solvable conformal field theory to argue that a given solution may be exact. In the present case of six dimensions, one should seek a string origin of the model. Indeed, one of the motivations for the construction of the BSS model was that it might be a worldvolume theory for a brane. In the last few years a great deal of progress has been made in understanding the types of branes that can arise in string/M-theory and in determining their worldvolume content. In particular, it has been shown that $(1,0)$ supersymmetric matter coupled Yang-Mills theories indeed arise in a configuration for configurations of branes involving NS fivebranes, $D6$ and $D8$ branes. The field content of the BSS model arises is a special case in which $D6$ branes stretch between NS fivebranes. For a summary of basic facts about this system, see for example, [27].

What is still lacking is the determination of the full nonlinear worldvolume action for the intersecting branes. That would provide an interesting nonlinear extension of the BSS model and an appropriate framework to study the brane within brane solitons.

It would also be interesting to formulate the matter coupled anomaly-free supergravity theories in six dimensions such that the classically gauge invariant and supersymmetric part of the action (or the equations of motion) is identified and the anomaly corrections are determined by means of the anomaly equations. A great deal of progress has been made in this direction in [8], but the task of determining the anomaly multiplet in terms of the supergravity plus matter multiplet fields remains to be completed. In addition to being relevant to intersecting branes and the resulting matter plus gauge theory dynamics, the study of the anomalies in six dimensions is also expected to shed light on the technically far more difficult problem of understanding the anomaly multiplet structure in ten dimensions.

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